

The Debt Simplification Problem

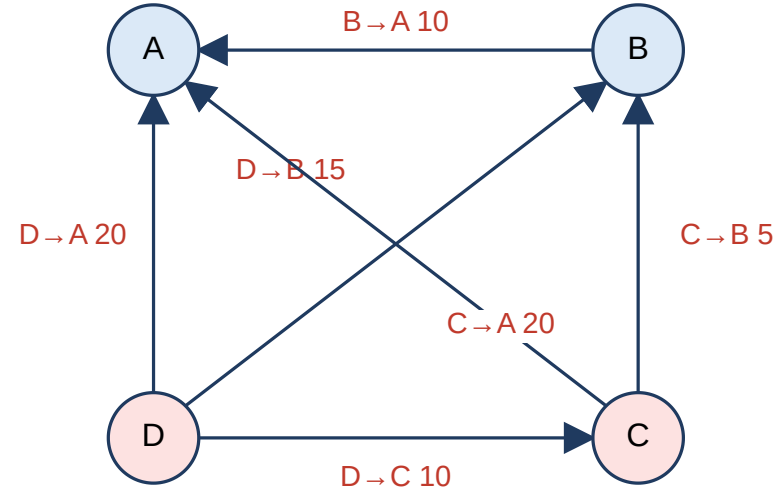
Splitting bills with graphs

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The trip

- Ala paid for the hotel
- Bartek paid for fuel
- Celina paid for dinner
- Darek paid for nothing

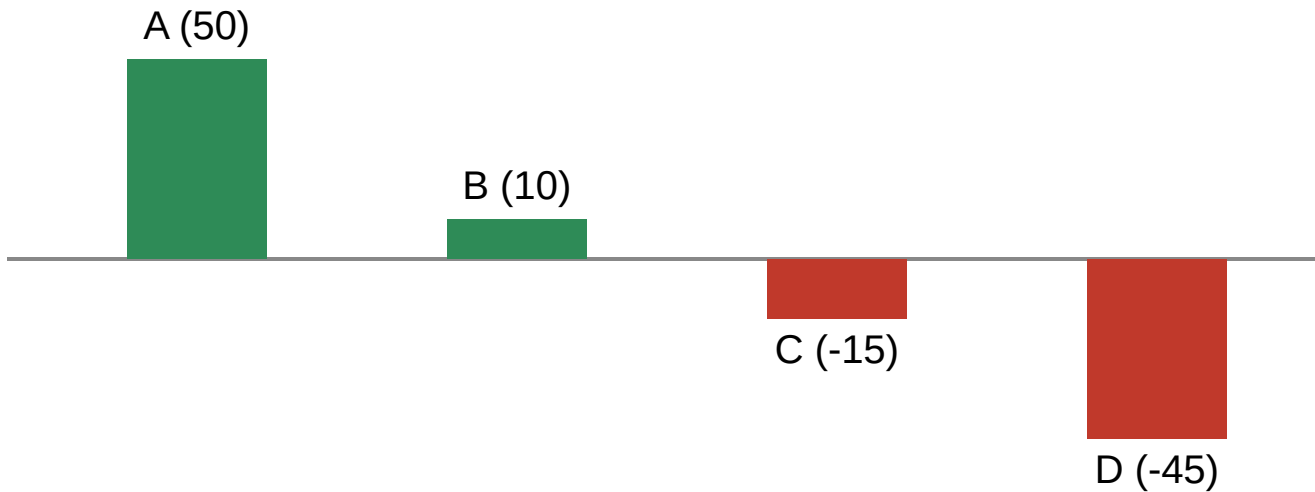
Result: **6 debts**, 4 people.



The one idea that matters

Nobody cares *who* they pay. Only the **net balance** matters.

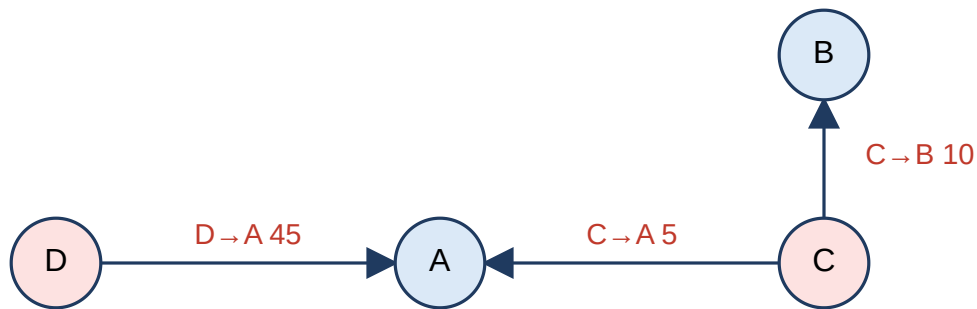
$$\text{net}(v) = \sum_{\text{in}} w - \sum_{\text{out}} w$$



Greedy: biggest debtor pays biggest creditor

Repeat until everyone is at zero:

1. pick the person with the **most negative** balance
2. pick the person with the **most positive** balance
3. transfer $\min(|\text{debt}|, \text{credit})$, one of them is settled



6 payments → **3.** At most $n - 1$ payments, always.

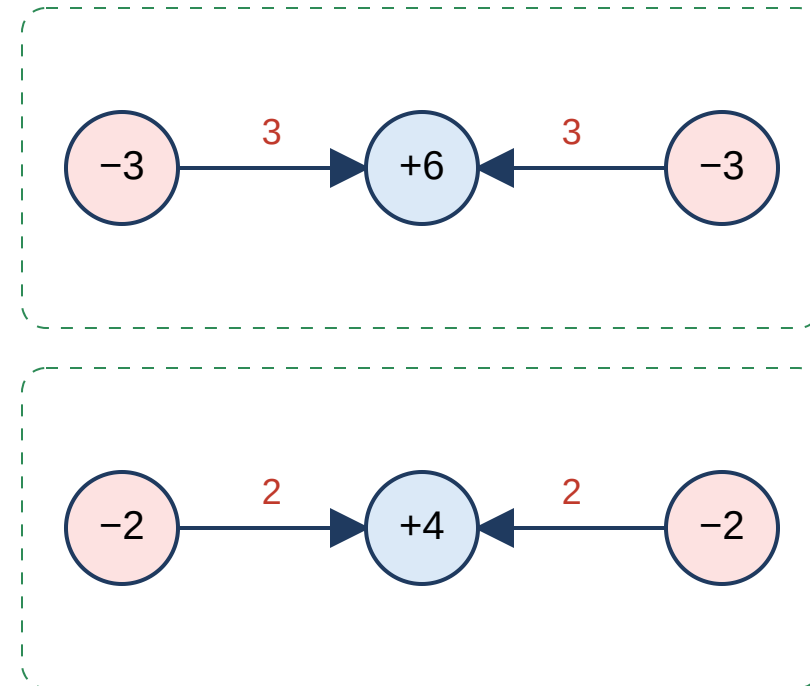
Greedy is not optimal

Balances: **+6 +4 -3 -3 -2 -2**

Greedy: **5** payments

Optimal: **4**

- $\{+6, -3, -3\}$ settles itself
- $\{+4, -2, -2\}$ settles itself
- two groups $\rightarrow n - 2$ payments



Why it is hard

Minimum payments = $n - (\text{max number of zero-sum groups})$

Deciding "is there a zero-sum group?" is **Subset Sum** → the problem is **NP-hard**.

Verhoeff, *Settling Multiple Debts Efficiently*, 2004

Exact solution: DP over subsets, $O(2^n \cdot n)$. Fine for a flat share, not for a company.

The twist

Splitwise shipped debt simplification.

Users **turned it off**.

"Why am I paying someone I never had dinner with?"

Thanks

Questions?

Slides: github.com/MrD4rkne/psics